Binomial Distributions

A binomial experiment is a probability experiment that satisfies these conditions.
1. The experiment has a fixed number of trials, where each trial is independent of the other trials.
2. There are only two possible outcomes of interest for each trial. Each outcome can be classified as a success (S) or failure (F).
3. The probability of success is the same for each trial.
4. The random variable \( x \) counts the number of successful trials.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>The number of trials</td>
</tr>
<tr>
<td>( p )</td>
<td>The probability of success in a single trial</td>
</tr>
<tr>
<td>( q )</td>
<td>The probability of failure in a single trial ( (q = 1 - p) )</td>
</tr>
<tr>
<td>( x )</td>
<td>The random variable represents a count of the number of successes in ( n ) trials: ( x = 0,1,2,3,4,\ldots,n )</td>
</tr>
</tbody>
</table>

From a standard deck of 52 cards, you pick a card, note whether it is a club or not, and replace the card. You repeat the experiment five times, so \( n = 5 \). The outcomes of each trial can be classified into two categories: S = selecting a club and F = selecting another suit. The probabilities of success and failure are

\[
p = \frac{1}{4} \quad q = \frac{3}{4}
\]

The random variable \( x \) represents the number of clubs selected in the five trials. So the possible values of the random variable are 0,1,2,3,4, and 5. For example, if \( x = 2 \), that means exactly 2 of the five cards drawn were clubs.
Determine whether each experiment is a binomial experiment. If it is, specify the values of \( n, p, \) and \( q \) and list the possible values of the random variable \( x \). If it is not explain why.

1. A certain surgical procedure has an 85% chance of success. A doctor performs the procedure on 8 patients. The random variable represents the number of successful surgeries.

2. A jar contains 5 red marbles, nine blue marbles, and six green marbles. You randomly select three marbles from the jar, without replacement. The random variable represents the number of red marbles.

3. You take a multiple-choice quiz that consists of 10 questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. The random variable represents the number of correct answers.

There are several ways to find the probability of \( x \) successes in \( n \) trials of a binomial experiment. One way is to use a tree diagram and the Multiplication Rule. Another way is to use the binomial probability formula.

### Binomial Probability Formula

In a binomial experiment, the probability of exactly \( x \) successes in \( n \) trials is

\[
P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}
\]

Note that the number of failures is \( n - x \).
Rotator cuff surgery has a 90% chance of success. The surgery is performed on three patients. Find the probability of the surgery being successful on exactly two patients.

Tree Diagram:

```
S = 0.9
F = 0.1

S
  0.9
  0.1
S
  0.9
  0.1
F
0.1
 S
  0.9
  0.1
S
  0.9
  0.1
F
0.1
 F
```

**Successes**

- $SSS = (0.9)(0.9)(0.9) = 0.729$
- $SSF = (0.9)(0.9)(0.1) = 0.081$
- $SFS = (0.9)(0.1)(0.9) = 0.081$
- $SFF = (0.9)(0.1)(0.1) = 0.009$
- $FSS = (0.1)(0.9)(0.9) = 0.081$
- $FSF = (0.1)(0.9)(0.1) = 0.009$
- $FFS = (0.1)(0.1)(0.9) = 0.009$
- $FFF = (0.1)(0.1)(0.1) = 0.001$

Add all the 2 successes together...$0.081 + 0.081 + 0.081 = 0.243$
Binomial Probability Formula:

\[ P(x) = \binom{n}{x} p^x q^{n-x} \]

For selecting exactly 3 clubs,

- \( n = 3 \)
- \( p = 0.9 \)
- \( q = 0.1 \)
- \( x = 2 \)

A card is selected from a standard deck and replaced. This experiment is repeated a total of five times. Find the probability of selecting exactly 3 clubs,
Binomial Probability Distribution

EXAMPLE 3 Page 204:

From example 3, we find that

\[ n = 7 \quad p = 0.46 \quad q = 0.54 \quad x = 0, 1, 2, 3, 4, 5, 6, \text{ and } 7 \]

\[ P(0) = \binom{7}{0} (0.46)^0 (0.54)^7 \approx 0.0134 \]
\[ P(1) = \binom{7}{1} (0.46)^1 (0.54)^6 \approx 0.0798 \]
\[ P(2) = \binom{7}{2} (0.46)^2 (0.54)^5 \approx 0.2040 \]
\[ P(3) = \binom{7}{3} (0.46)^3 (0.54)^4 \approx 0.2897 \]
\[ P(4) = \binom{7}{4} (0.46)^4 (0.54)^3 \approx 0.2468 \]
\[ P(5) = \binom{7}{5} (0.46)^5 (0.54)^2 \approx 0.1261 \]
\[ P(6) = \binom{7}{6} (0.46)^6 (0.54)^1 \approx 0.0358 \]
\[ P(7) = \binom{7}{7} (0.46)^7 (0.54)^0 = 1 (0.46)^7 (0.54)^0 \approx 0.0044 \]

All probabilities are between 0 and 1. The sum of the probabilities equals 1.
To use the Binomial Probability on the calculator:

2ND DISTR scroll to A binompdf( ENTER n (#trials)
p(prob of success) x value: # of successes

67% of US adults consider air conditioning a necessity. You randomly select 100 adults. What is the probability that exactly 75 adults consider air conditioning a necessity?

2ND DISTR scroll to A binompdf( ENTER Trials: 100
p: .67 x value: 75 Enter Enter 0.0201

A survey found that 34% of US adults have hidden purchases from their spouses. You randomly select 200 adults with spouses. What is the probability that exactly 68 of them have hidden purchases from their spouses?
Finding Binomial Probabilities Using Formulas

A survey of US adults found that 62% of women believe that there is a link between playing violent video games and teens exhibiting violent behavior. You randomly select four US women and ask them whether they believe there is a link between playing violent video games and teens exhibiting violent behavior. Find (a) exactly two of them respond yes, (b) at least two of them respond yes, and (c) fewer than two of them respond yes.

\( n = 4 \quad p = 0.62 \quad q = 0.38 \)

(a) \( x = 2 \)

\( P(0) = \binom{4}{0} (0.62)^0 (0.38)^4 = 1(0.62)^0 (0.38)^4 \approx 0.021 \)

\( P(1) = \binom{4}{1} (0.62)^1 (0.38)^3 = 4(0.62)^1 (0.38)^3 \approx 0.136 \)

\( P(2) = \binom{4}{2} (0.62)^2 (0.38)^2 = 6(0.62)^2 (0.38)^2 \approx 0.333 \)

\( P(3) = \binom{4}{3} (0.62)^3 (0.38)^1 = 4(0.62)^3 (0.38)^1 \approx 0.362 \)

\( P(4) = \binom{4}{4} (0.62)^4 (0.38)^0 = 1(0.62)^4 (0.38)^0 \approx 0.148 \)

(a) \( P(2) = 0.333 \)

(b) \( P \left( x \geq 2 \right) = 0.333 + 0.362 + 0.148 \approx 0.843 \)

(c) \( P(x < 2) = 0.136 + 0.021 = 0.157 \) or \( P \left( x \geq 2 \right) = 1 - 0.843 \)
Finding a Binomial Probability Using a Table

EXAMPLE 6 Page 207:
Finding a binomial probability can be a tedious process using the binomial probability formula. To make things easier, you can use a binomial probability table. This table is TABLE 2 in Appendix B on page A8.

Using the portion of the table on page 207, about 10% of workers (16 and older) in the United States commute to their jobs by carpooling. You randomly select eight workers. What is the probability that exactly four of them carpool to work?

Look at the table and find the probability of success, \( p = 0.10 \), then slide down to the \( n = 8 \) and in that category, find 4. In this example, it is 0.005. The probability of exactly 4 of the eight workers carpool is 0.005. Because this is less than 0.05, this is an unusual event.

TRY:

About 55% of all small businesses in the US have a website. You randomly select 10 small businesses. What is the probability that exactly 4 will have a website?
About 60% of cancer survivors are ages 65 and older. You randomly select 6 cancer survivors and ask them whether they are 65 years of age or older. Construct a probability distribution for the random variable $x$. Then graph the distribution.

Using the table in appendix B, we get:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.004</td>
</tr>
<tr>
<td>1</td>
<td>0.037</td>
</tr>
<tr>
<td>2</td>
<td>0.138</td>
</tr>
<tr>
<td>3</td>
<td>0.276</td>
</tr>
<tr>
<td>4</td>
<td>0.311</td>
</tr>
<tr>
<td>5</td>
<td>0.187</td>
</tr>
<tr>
<td>6</td>
<td>0.047</td>
</tr>
</tbody>
</table>

![Graph showing the probability distribution](image)
Mean, Variance, and Standard Deviation

Population Parameters of a Binomial Distribution

Mean: $\mu = np$
Variation: $\sigma^2 = npq$
Standard deviation: $\sigma = \sqrt{npq}$

In Pittsburgh, Pennsylvania, about 56% of the days in a year are cloudy. Find the mean variance, and standard deviation for the number of cloudy days during the month of June. Interpret the results and determine any unusual values.

$$n = 30 \quad p = 0.56 \quad q = 0.44$$

$$\mu = np = 30(0.56) \approx 16.8$$
$$\sigma^2 = npq = 30(0.56)(0.44) \approx 7.4$$
$$\sigma = \sqrt{npq} = \sqrt{7.4} \approx 2.7$$

On average there are about 16.8 cloudy days during the month of June. Because $16.8 - 2(2.7) = 11.4$ and $16.8 + 2(2.7) = 22.2$, a June with 11.4 or fewer cloudy days or a June with 22.2 or more cloudy days would be considered unusual.
P. 210 7 - 21 odds, 25, 27, 29